PROBLEMS

4.1 Dilution of power plant plumes

Match each power plant plume (1-4) to the corresponding atmospheric lapse rate (A-D, solid lines; the dashed line is the adiabatic lapse rate $\Gamma$). Briefly comment on each case.
4.2 Short questions on atmospheric transport

1. Pollutants emitted in the United States tend to be ventilated by vertical transport in summer and by horizontal transport in winter. Explain this seasonal difference.

2. Solar heating of the Earth’s surface facilitates not only the upward but also the downward transport of air pollutants. Explain.

3. A monitoring station measures the vertical concentration profiles of a pollutant emitted at a constant and uniform rate at the surface. The profiles measured on two successive days are shown below:

Which of these two profiles is consistent with a one-dimensional turbulent diffusion parameterization of turbulence? How would you explain the other profile?

4. A power plant in a city discharges a pollutant continuously from a 200-m tall stack. At what time of day would you expect the surface air concentrations of the pollutant in the city to be highest?

5. In a conditionally unstable atmosphere (-dT/dz < Γ_w), is a cloudy air parcel stable or unstable with respect to sinking motions? Can these "downdraft" motions lead to rapid vertical transport of air from the upper to the lower troposphere? Briefly explain.

6. For a gas that is well mixed in the atmosphere, is there any turbulent transport flux associated with turbulent motions? Briefly explain.
4.3 Seasonal motion of the ITCZ

The mean latitude of the ITCZ varies seasonally from 5°S in January to 10°N in July, following the orientation of the Earth relative to the Sun. By using a two-box model for transfer of air between the northern and the southern hemispheres, with the ITCZ as a moving boundary between the two boxes, calculate the fraction of hemispheric mass transferred by this process from one hemisphere to the other over the course of one year. Does this process make an important contribution to the overall interhemispheric exchange of air?

4.4 A simple boundary layer model

We construct a simple model for diurnal mixing in the planetary boundary layer (PBL) by dividing the PBL vertically into two superimposed domains: (1) the mixed layer and (2) the remnant PBL (see figure below). These two domains are separated by an inversion, and a second inversion caps the remnant PBL. We assume that the domains are individually well mixed and that there is no vertical exchange across the inversions.

1. Provide a brief justification for this model, and for the diurnal variation in the sizes of the two domains. Why is there a mixed layer at night? (Hint: buoyancy is not the only source of vertical turbulent mixing).

2. Consider an inert pollutant X emitted from the surface with a constant emission flux beginning at \( t = 0 \) (midnight). Plot the change in the concentration of X from \( t = 0 \) to \( t = 24 \) hours in domains (1) and (2), starting from zero concentrations at \( t = 0 \) in both domains.

4.5 Breaking a nighttime inversion

A town suffers from severe nighttime smoke pollution during the winter months because of domestic wood burning and strong temperature inversions. Consider the following temperature profile measured at dawn:
We determine in this problem the amount of solar heating necessary to break the inversion and ventilate the town.

1. Show on the figure the minimum temperature rise required to ventilate the town.

2. Show that the corresponding heat input per unit area of surface is \( Q = 2.5 \times 10^6 \) J m\(^{-2}\). Use \( \rho = 1 \) kg m\(^{-3}\) for the density of air and \( C_p = 1 \times 10^3 \) J kg\(^{-1}\) K\(^{-1}\) for the specific heat of air at constant pressure.

3. Solar radiation heats the surface after sunrise, and the resulting heat flux \( F \) to the atmosphere is approximated by

\[
F = F_{max} \cos \left( \frac{2\pi(t - t_{noon})}{\Delta t} \right) \quad 6 \text{ a.m.} < t < 6 \text{ p.m.}
\]

where \( F_{max} = 300 \) W m\(^{-2}\) is the maximum flux at \( t_{noon} = 12 \) p.m., and \( \Delta t = 24 \) hours. At what time of day will the town finally be ventilated?

4. 6 Wet convection

Consider the following observed temperature profile:
1. Identify stable and unstable regions in the profile. Briefly explain.

2. Consider an air parcel rising from A to B and forming a cloud at point B. Explain how cloud formation allows further rise of the air parcel. Assuming a wet adiabatic lapse rate $\Gamma_w = 6 \text{ K km}^{-1}$, calculate the altitude to which the air parcel will rise before it becomes stable relative to the surrounding atmosphere.

3. Conclude as to the effect of cloud formation for the ventilation of pollution released at the surface.

4. **Scavenging of water in a thunderstorm**

Consider a tropical thunderstorm in which air saturated with water vapor at 25 °C and 1 km altitude is brought adiabatically to 15 km altitude in a cloud updraft. Assume a mean lapse rate $\Gamma_W = 4 \text{ K km}^{-1}$ in the updraft. Further assume that water vapor is immediately precipitated upon condensation, a reasonable assumption since the amount of suspended cloudwater in an air parcel is small relative to the amount of water vapor (problem 1.2). When the air exits the cloud updraft at 15 km altitude, what fraction of its initial water vapor has been removed by precipitation?

4. **Global source of methane**

Emission of methane to the atmosphere is largely biogenic and the individual sources are difficult to quantify. However, one can use a simple mass balance approach to calculate the global source.

1. Methane is removed from the troposphere by oxidation, and the corresponding lifetime of methane is known to be 9 years (as will be seen in chapter 11). Based on this lifetime, would you expect methane to be well mixed in the troposphere? Briefly explain.

2. The present-day methane concentration in the troposphere is 1700 ppbv and is rising at the rate of 10 ppbv yr$^{-1}$. Using a mass balance equation for methane in the troposphere, show that the present-day emission of methane is $E = 3.0 \times 10^{13}$ moles per year. For this calculation, take 150 hPa as the top of the troposphere and neglect transport of methane to the stratosphere.

3. We now refine our estimate by accounting for the chemical loss of methane in the stratosphere.
3.1. The mixing ratio $C$ of methane above the tropopause (altitude $z_t$) decreases exponentially with altitude, with a scale height $h = 60$ km. Using a turbulent diffusion formulation for the vertical flux and assuming steady state for methane in the stratosphere, show that:

$$-A K_z n_a \frac{dC}{dz}_{\text{tropopause}} = L_{\text{strat}}$$

where the turbulent diffusion coefficient $K_z$, the air density $n_a$, and the methane mixing ratio are evaluated just above the tropopause; $A$ is the surface area of the Earth; and $L_{\text{strat}}$ is the total chemical loss of methane in the stratosphere.

3.2. Calculate $L_{\text{strat}}$ assuming a turbulent diffusion coefficient $K_z = 7 \times 10^3$ cm$^2$ s$^{-1}$ and an air density $n_a = 5 \times 10^{18}$ molecules cm$^{-3}$ just above the tropopause. From this result, derive an improved estimate of the present-day emission of methane.

4.9 Role of molecular diffusion in atmospheric transport

The molecular diffusion coefficient $D$ of air increases with altitude as the mean free path between molecular collisions increases. One can show that $D$ varies inversely with pressure:

$$D = D_o \frac{P_o}{P}$$

where $P_o$ is the pressure at sea level and $D_o = 0.2$ cm$^2$ s$^{-1}$ is the molecular diffusion coefficient at sea level.

1. Calculate the average time required for a molecule to travel 1 m by molecular diffusion at sea level, at 10 km altitude, and at 100 km altitude.

2. At what altitude does molecular diffusion become more important than turbulent diffusion as a mechanism for atmospheric transport? Assume a turbulent diffusion coefficient $K_z = 1 \times 10^5$ cm$^2$ s$^{-1}$ independent of altitude.

4.10 Vertical transport near the surface

Vertical transport near the surface is often modeled with a turbulent diffusion coefficient $K_z = \alpha z$ where $\alpha$ is a constant and $z$ is altitude. Consider a species subsiding in the atmosphere and removed solely by reaction at the Earth's surface. Show that the mixing ratio of this species near the surface increases logarithmically with altitude. (You may assume steady state conditions and neglect changes in air density with altitude).