CHAPTER 2. ATMOSPHERIC PRESSURE

2.1 MEASURING ATMOSPHERIC PRESSURE

The *atmospheric pressure* is the weight exerted by the overhead atmosphere on a unit area of surface. It can be measured with a mercury barometer, consisting of a long glass tube full of mercury inverted over a pool of mercury:

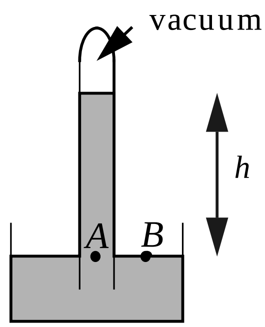

Figure 2-1 Mercury barometer

When the tube is inverted over the pool, mercury flows out of the tube, creating a vacuum in the head space, and stabilizes at an equilibrium height h over the surface of the pool. This equilibrium requires that the pressure exerted on the mercury at two points on the horizontal surface of the pool, A (inside the tube) and B (outside the tube), be equal. The pressure P_A at point A is that of the mercury column overhead, while the pressure P_B at point B is that of the atmosphere overhead. We obtain P_A from measurement of h:

$$P_A = \rho_{Hg} g h \qquad (2.1)$$

where ρ_{Hg} = 13.6 g cm^{-3} is the density of mercury and g = 9.8 m s^{-2} is the acceleration of gravity. The mean value of h measured at sea level is 76.0 cm, and the corresponding atmospheric pressure is 1.013x10^5 kg m^{-1} s^{-2} in SI units. The SI pressure unit is called the *Pascal* (Pa); 1 Pa = 1 kg m^{-1} s^{-2}. Customary pressure units are the *atmosphere* (atm) (1 atm = 1.013x10^5 Pa), the *bar* (b) (1 b = 1x10^5 Pa), the *millibar* (mb) (1 mb = 100 Pa), and the *torr* (1 torr = 1 mm Hg = 134 Pa). The use of millibars is slowly giving way to the equivalent SI unit of hectoPascals (hPa). The mean atmospheric pressure at sea level is given equivalently as P = 1.013x10^5 Pa = 1013 hPa = 1013 mb = 1 atm = 760 torr.

2.2 MASS OF THE ATMOSPHERE

The global mean pressure at the surface of the Earth is P_S = 984 hPa, slightly less than the mean sea-level pressure because of the elevation of land. We deduce the total mass of the atmosphere m_a:

$$m_a = \frac{4\pi R^2 P_S}{g} = 5.2x10^{18} kg \qquad (2.2)$$

where R = 6400 km is the radius of the Earth. The total number of moles of air in the atmosphere is $N_a = m_a/M_a$ = $1.8\text{x}10^{20}$ moles.

Exercise 2-1. Atmospheric CO_2 concentrations have increased from 280 ppmv in preindustrial times to 365 ppmv today. What is the corresponding increase in the mass of atmospheric carbon? Assume CO_2 to be well mixed in the atmosphere.

Answer. We need to relate the mixing ratio of CO_2 to the corresponding mass of carbon in the atmosphere. We use the definition of the mixing ratio from equation *(1.3)*,

$$C_{CO2} = \frac{n_{CO2}}{n_a} = \frac{N_C}{N_a} = \frac{M_a}{M_C} \cdot \frac{m_C}{m_a}$$

where N_C and N_a are the total number of moles of carbon (as CO_2) and air in the atmosphere, and m_C and m_a are the corresponding total atmospheric masses. The second equality reflects the assumption that the CO_2 mixing ratio is uniform throughout the atmosphere, and the third equality reflects the relationship $N = m/M$. The change Δm_C in the mass of carbon in the atmosphere since preindustrial times can then be related to the change ΔC_{CO2} in the mixing ratio of CO_2. Again, always use SI units when doing numerical calculations (this is your last reminder!):

$$\Delta m_C = m_a \frac{M_C}{M_a} \cdot \Delta C_{CO2} = 5.2\text{x}10^{18} \cdot \frac{12\text{x}10^{-3}}{29\text{x}10^{-3}} \cdot (365\text{x}10^{-6} - 280\text{x}10^{-6})$$

$$= 1.8\text{x}10^{14} \text{ kg} = 180 \text{ billion tons!}$$

2.3 VERTICAL PROFILES OF PRESSURE AND TEMPERATURE

Figure 2-2 shows typical vertical profiles of pressure and temperature observed in the atmosphere. Pressure decreases exponentially with altitude. The fraction of total atmospheric weight located above altitude z is $P(z)/P(0)$. At 80 km altitude the atmospheric pressure is down to 0.01 hPa, meaning that 99.999% of the atmosphere is below that altitude. You see that the atmosphere is of relatively thin vertical extent. Astronomer Fred Hoyle once said, "Outer space is not far at all; it's only one hour away by car if your car could go straight up!"

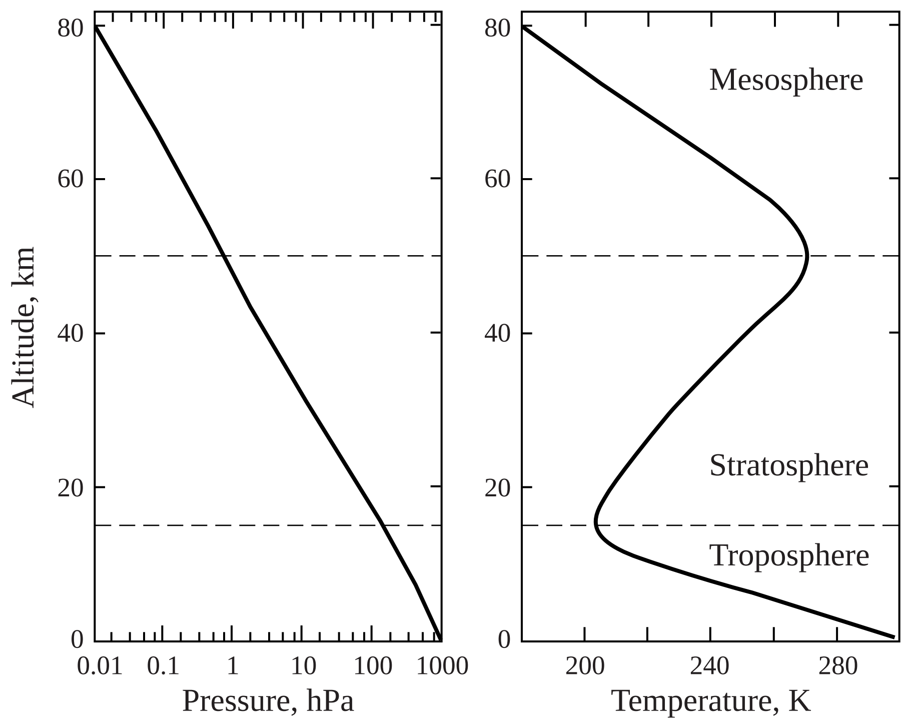

Figure 2-2 Mean pressure and temperature vs. altitude at 30^{o}N, March

Atmospheric scientists partition the atmosphere vertically into domains separated by reversals of the temperature gradient, as shown in Figure 2-2. The *troposphere* extends from the surface to 8-18 km altitude depending on latitude and season. It is characterized by a decrease of temperature with altitude which can be explained simply though not quite correctly by solar heating of the surface (we will come back to this issue in chapters 4 and 7). The *stratosphere* extends from the top of the troposphere (the *tropopause)* to about 50 km altitude (the *stratopause*) and is characterized by an increase of temperature with altitude due to absorption of solar radiation by the ozone layer (problem 1. 3). In the *mesosphere*, above the ozone layer, the temperature decreases again with altitude. The mesosphere extends up to 80 km (*mesopause*) above which lies the *thermosphere* where temperatures increase again with altitude due to absorption of strong UV solar radiation by N_2 and O_2. The troposphere and stratosphere account together for 99.9% of total atmospheric mass and are the domains of main interest from an environmental perspective.

Exercise 2-2 What fraction of total atmospheric mass at 30°N is in the troposphere? in the stratosphere? Use the data from Figure 2-2.

Answer. The troposphere contains all of atmospheric mass except for the fraction *P(tropopause)/P(surface)* that lies above the tropopause. From Figure 2-2 we read *P(tropopause)* = 100 hPa, *P(surface)* = 1000 hPa. The fraction F_{trop} of total atmospheric mass in the troposphere is thus

$$F_{trop} = 1 - \frac{P(tropopause)}{P(0)} = 0.90$$

The troposphere accounts for 90% of total atmospheric mass at 30°N (85% globally).

The fraction F_{strat} of total atmospheric mass in the stratosphere is given by the fraction above the tropopause, *P(tropopause)/P(surface)*, minus the fraction above the stratopause, *P(stratopause)/P(surface)*. From Figure 2-2 we read *P(stratopause)* = 0.9 hPa, so that

$$F_{strat} = \frac{P(tropopause) - P(stratopause)}{P(surface)} = 0.099$$

The stratosphere thus contains almost all the atmospheric mass above the troposphere. The mesosphere contains only about 0.1% of total atmospheric mass.

2.4 BAROMETRIC LAW

We will examine the factors controlling the vertical profile of atmospheric temperature in chapters 4 and 7. We focus here on explaining the vertical profile of pressure. Consider an elementary slab of atmosphere (thickness *dz*, horizontal area *A*) at altitude *z*:

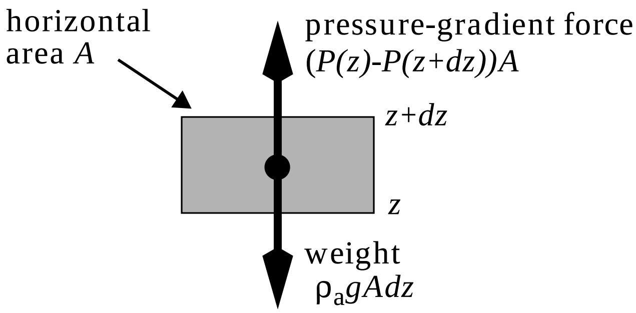

Figure 2-3 Vertical forces acting on an elementary slab of atmosphere

The atmosphere exerts an upward pressure force $P(z)A$ on the bottom of the slab and a downward pressure force $P(z+dz)A$ on the top of the slab; the net force, $(P(z)-P(z+dz))A$, is called the *pressure-gradient force*. Since $P(z) > P(z+dz)$, the pressure-gradient force is directed upwards. For the slab to be in equilibrium, its weight must balance the pressure-gradient force:

$$\rho_a gAdz = (P(z) - P(z+dz))A \tag{2.3}$$

Rearranging yields

$$\frac{P(z+dz) - P(z)}{dz} = -\rho_a g \tag{2.4}$$

The left hand side is dP/dz by definition. Therefore

$$\frac{dP}{dz} = -\rho_a g \tag{2.5}$$

Now, from the ideal gas law,

$$\rho_a = \frac{PM_a}{RT} \tag{2.6}$$

where M_a is the molecular weight of air and T is the temperature. Substituting *(2.6)* into *(2.5)* yields:

$$\frac{dP}{P} = -\frac{M_a g}{RT}dz \tag{2.7}$$

We now make the simplifying assumption that T is constant with altitude; as shown in Figure 2-2, T varies by only 20% below 80 km. We then integrate *(2.7)* to obtain

$$\ln P(z) - \ln P(0) = -\frac{M_a g}{RT} z \quad (2.8)$$

which is equivalent to

$$P(z) = P(0)\exp\left(-\frac{M_a g}{RT} z\right) \quad (2.9)$$

Equation *(2.9)* is called the *barometric law*. It is convenient to define a sc*ale height* H for the atmosphere:

$$H = \frac{RT}{M_a g} \quad (2.10)$$

leading to a compact form of the Barometric Law:

$$P(z) = P(0)e^{-\frac{z}{H}} \quad (2.11)$$

For a mean atmospheric temperature T = 250 K the scale height is H = 7.4 km. The barometric law explains the observed exponential dependence of P on z in Figure 2-2; from equation *(2.11)*, a plot of z vs. ln P yields a straight line with slope -H (check out that the slope in Figure 2-2 is indeed close to -7.4 km). The small fluctuations in slope in Figure 2-2 are caused by variations of temperature with altitude which we neglected in our derivation.

The vertical dependence of the air density can be similarly formulated. From *(2.6)*, ρ_a and P are linearly related if T is assumed constant, so that

$$\rho_a(z) = \rho_a(0)e^{-\frac{z}{H}} \quad (2.12)$$

A similar equation applies to the air number density n_a. For every H rise in altitude, the pressure and density of air drop by a factor e = 2.7; thus H provides a convenient measure of the thickness of the atmosphere.

In calculating the scale height from *(2.10)* we assumed that air behaves as a homogeneous gas of molecular weight M_a = 29 g mol^{-1}. Dalton's law stipulates that each component of the air mixture must behave as if it were alone in the atmosphere. One might then expect different components to have different scale heights determined by their molecular weight. In particular, considering the difference in molecular weight between N_2 and O_2, one might expect the O_2 mixing ratio to decrease with altitude. However, gravitational separation of the air mixture takes place by molecular diffusion, which is considerably slower than turbulent vertical mixing of air for altitudes below 100 km (problem 4. 9). Turbulent mixing thus maintains a homogeneous lower atmosphere. Only above 100 km does significant gravitational separation of gases begin to take place, with lighter gases being enriched at higher altitudes. During the debate over the harmful effects of chlorofluorocarbons (CFCs) on stratospheric ozone, some not-so-reputable scientists claimed that CFCs could not possibly reach the stratosphere because of their high molecular weights and hence low scale heights. In reality, turbulent mixing of air ensures that CFC mixing ratios in air entering the stratosphere are essentially the same as those in surface air.

Exercise 2-3 The cruising altitudes of subsonic and supersonic aircraft are 12 km and 20 km respectively. What is the relative difference in air density between these two altitudes?

Answer. Apply (2.12) with z_1 = 12 km, z_2 = 20 km, H = 7.4 km:

$$\frac{\rho(z_2)}{\rho(z_1)} = \frac{e^{-\frac{z_2}{H}}}{e^{-\frac{z_1}{H}}} = e^{-\frac{(z_2 - z_1)}{H}} = 0.34$$

The air density at 20 km is only a third of that at 12 km. The high speed of supersonic aircraft is made possible by the reduced air resistance at 20 km.

2.5 THE SEA-BREEZE CIRCULATION

An illustration of the Barometric Law is the sea-breeze circulation commonly observed at the beach on summer days (Figure 2-4). Consider a coastline with initially the same atmospheric temperatures and pressures over land (L) and over sea (S). Assume that there is initially no wind. In summer during the day the land surface is heated to a higher temperature than the sea. This difference is due in part to the larger heat capacity of the sea, and in part to the consumption of heat by evaporation of water.

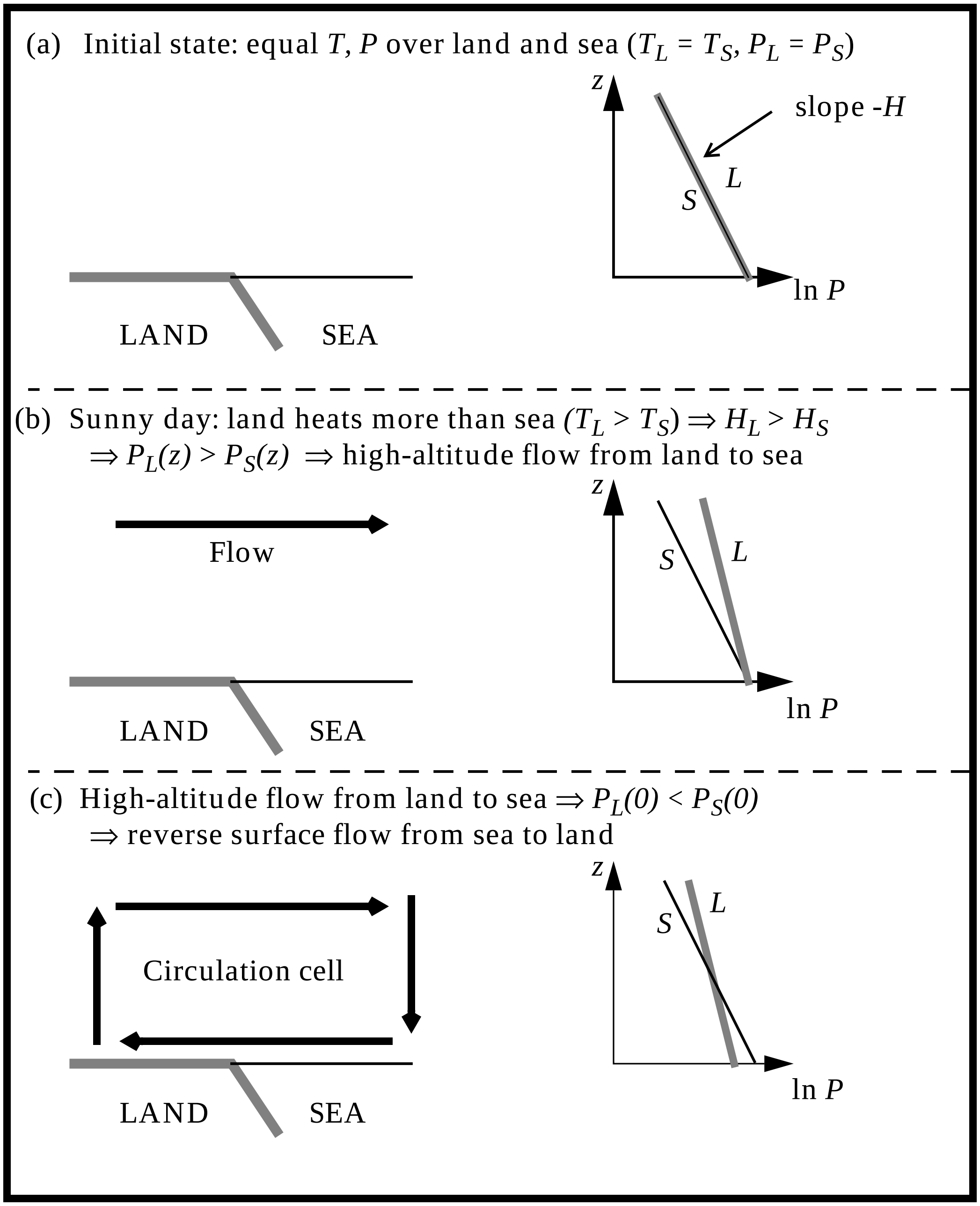

Figure 2-4 The sea-breeze circulation

As long as there is no flow of air between land and sea, the total air columns over each region remain the same so that at the surface $P_L(0) = P_S(0)$. However, the higher temperature over land results in a larger atmospheric scale height over land ($H_L > H_S$), so that above the surface $P_L(z) > P_S(z)$ (Figure 2-4). This pressure difference causes the air to flow from land to sea, decreasing the mass of the air column over the land; consequently, at the surface, $P_L(0) < P_S(0)$ and the wind blows from sea to land (the familiar "sea breeze"). Compensating vertical motions result in the circulation cell shown in Figure 2-4. This cell typically extends ~10 km horizontally across the coastline and ~1 km vertically. At night a reverse circulation is frequently observed (the land breeze) as the land cools faster than the sea.