Reduced-rank Jacobians: Decreasing the computational cost of high resolution analytic inversions

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Introduction

- The Tropospheric Monitoring Instrument (TROPOMI) provides daily, global column methane concentration retrievals in 7 x 7 km² pixels.
- Methane emissions can be inferred at high-resolution by inverting these observations with a chemical transport model (CTM). A Bayesian inversion accounts for errors and, when the CTM is linear, gives a closed-form solution for the posterior emissions estimate.
- The computational cost of this solution is limited by the dimension \( n \) of the emission (state) vector due largely to the cost of characterizing the linear relationship between emissions and observations, given by the Jacobian \( K \).
- We propose an iterative method of constructing \( K \) that reduces computational cost by finding the model response to the eigenvectors of the information content rather than individual state vector elements.

Reduced-rank Jacobians

- \( K \) is often constructed by perturbing each of the \( n \) state vector elements and finding the model response, requiring \( n+1 \) model runs.
- Not all state vector elements contribute equal information to the inversion. An eigendecomposition of the averaging kernel \( A \) gives the patterns of information content, and \( k \ll n \) eigenvectors explain most of the variance of the inverse system.
- Perturbing \( k \ll n \) eigenvectors reduces the number of model runs needed to construct \( K \). This yields a reduced-dimension Jacobian, which must be transformed to the full-dimension space.
- \( A \) is a function of \( K \), so it is necessary to iteratively update \( K \) from an initial estimate.

Algorithm

- We propose an iterative method to construct \( K \) that refines an initial estimate in areas with high information content:
  i. Initialize the Jacobian \( K_0 \).
  ii. Calculate \( \Pi, \Gamma, \Gamma^* \).
  iii. Perturb \( k \ll n \) eigenvectors and find the reduced-dimension Jacobian \( K_{\text{red}} \).
  iv. Find \( K_{i+1} \) by minimizing \( ||K_i - K_{i+1}|| \) subject to \( ||\Gamma^* K_i - K_{\text{red}}|| = 0 \).
  v. Return to ii and iterate until convergence.

Conclusions

- Reduced-rank Jacobians will reduce the computational cost of high-resolution analytic Bayesian inversions by decreasing the number of perturbation model runs.
- The reduced-rank Jacobian can be used in reduced-rank inversions to further reduce computational cost.
- An a posteriori filter can be applied to account for errors in the reduced-rank Jacobian.

References