Reduced-Cost Construction of Jacobian Matrices for High-Resolution Inverse Modeling

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TROPOMI provides daily, global retrievals of atmospheric methane columns.
Satellite retrievals can be used in inverse frameworks to improve constraints on emissions estimates.

Emissions estimate $(x_A, S_A)$

Modeled observations $(Kx_A + c)$

Forward model

Improved emissions estimate $(\hat{x}, \hat{S})$

Observations $(y, S_0)$

Minimize cost function
<table>
<thead>
<tr>
<th>Minimization method:</th>
<th>Variational</th>
<th>Analytic</th>
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<tbody>
<tr>
<td>Computational cost is not limited by resolution</td>
<td>O</td>
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<td>Finds true minimum of shallow cost function</td>
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<td>O</td>
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<tr>
<td>Characterizes posterior error and information content</td>
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<td>O</td>
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<td>Sensitivity tests require no significant additional computational cost</td>
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<td>O</td>
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The computational cost of an analytic inversion is limited by constructing the Jacobian $\mathbf{K}$.
Analytic inversion of July 2009 GOSAT observations over North America at 1° x 1.25° requires 2,099 model runs.
Analytic solution of the inversion shows that not all grid cells are equally well constrained

Averaging Kernel

\[ n = 2,098 \]
\[ \text{DOFS} = 81 \]
Perturbing the dominant patterns of information content would require $k < n$ model runs.

- **emissions estimate**
- **modeled observations**

**Forward model**

$\Delta x$, $\Delta y$

$k$ perturbations, $k$ model runs
Initialize the Jacobian

Find the dominant patterns of information content

Perturb those patterns in the forward model

Transform the resulting Jacobian to the original resolution
Find the dominant patterns of information content

Initialize the Jacobian

Perturb those patterns in the forward model

Transform the resulting Jacobian to the original resolution
Initialize the Jacobian

Find the dominant patterns of information content

Perturb those patterns in the forward model

Transform the resulting Jacobian to the original resolution
Can the low-rank Jacobian reproduce the true posterior solution?

2,099 model runs
2,098 grid boxes
81 DOFS
The low-rank Jacobian solves the inversion accurately in the grid cells with highest information content.

- 306 model runs
- 622 grid boxes
- 49 DOFS
The low-rank Jacobian solves the inversion accurately in the grid cells with highest information content.
Low-rank Jacobian approximations significantly reduce the computational cost of analytic Bayesian inversions

• Perturbing the primary directions of information content can decrease the computational cost of constructing the Jacobian by an order of magnitude.

• The resulting Jacobian produces a posterior solution that is accurate in areas with sufficient information content.
Initialize the Jacobian

Find the dominant patterns of information content

Perturb those patterns in the forward model

Transform the resulting Jacobian to the original resolution
Initial estimate: eigenvectors
Initial estimate: eigenvalues
Final estimate: eigenvectors
Final estimate: eigenvalues
Error in approximate mean

Approximate Posterior Mean - True Mean
Optimized grid cells with incorrect sign
Errors in approximate mean vs. posterior error
Error vs. information content