Using satellite data in joint CO$_2$-CO inversion to improve CO$_2$ flux estimates

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Submitted to ACP
Why $\text{CO}_2 : \text{CO}$?

- $\text{CO}$ and $\text{CO}_2$ share common combustion sources and transport, indicating cross correlation in concentration and error.
- $\text{CO}$ has stronger gradient, is more sensitive to transport error, can provide additional constraint.
- $\text{CO}$ is relatively easy to measure from space; Multiple validated data sets have been used for $\text{CO}$ source inversion.
- Joint $\text{CO} – \text{CO}_2$ inversion using aircraft data in Asian outflow showed substantial improvement over $\text{CO}_2 –$ only inversion.

[Palmer et al., 2006]
Joint CO$_2$ – CO inversion

**Observation Vector:**
- Column concentration

**State Vector:**
- Sources

**Cost function**

\[ J(x) = (y - Kx)^T S_{\Sigma}^{-1} (y - Kx) + (x - x_a)^T S_a^{-1} (x - x_a) \]

**Observational error covariance**

**A priori error covariance**

Coupling between CO$_2$ and CO occurs through off-diagonal elements in error covariance matrices $S$

\[
S = \begin{pmatrix}
S_{CO} & \text{cov}(\varepsilon_{CO}, \varepsilon_{CO_2}) \\
\text{cov}(\varepsilon_{co}, \varepsilon_{co_2}) & S_{CO_2}
\end{pmatrix}
\]

\[
\text{cov}(\varepsilon_{co}, \varepsilon_{co_2}) = r \sqrt{\text{var}(\varepsilon_{co})} \sqrt{\text{var}(\varepsilon_{co_2})}
\]

**Correlation coefficient**

- Off diagonal elements increase information content
- Negligible correlation in $S_a$ due to CO emission factor uncertainty
Components of observational error

\[ \mathcal{E}_\Sigma = \mathcal{E}_I + \mathcal{E}_R + \mathcal{E}_M \]

- Observational error
- Instrument error
- Representation error

Model error
- dominant
- important

Covariance structure comes from the model error
Calculating model transport error correlation $r_M$

1. Paired model: GEOS4 – GEOS5
   (same meteorological year, same sources/sinks)

The same can be performed for GEOS3 – GEOS4 pair
Model error correlation 1:30 PM, no averaging kernel

Large scale error correlation patterns are robust

Error correlation is different from concentration correlation
2. Model error correlation from paired forecast (NMC) method

GEOS5 ARCTAS 48h – 24h forecast of column CO and CO$_2$

July 2008

the same large scale pattern as before
Analytical inversion results for Europe with globally **uniform** model error correlation and no instrument or representation errors

- Substantial improvements when $\Gamma_M > 0.6$

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**A posteriori CO₂ flux uncertainty**

- 14 days of pseudo column CO and CO₂ data from GEOS-Chem 2x2.5 simulation sampled from A-train orbit
- With OCO-like averaging kernels for column CO and CO₂
- Substantial improvements when $\Gamma_M > 0.6$
Results with **model error correlation map** and no instrument or representation errors

Combustion Biosphere

Dec 2005

July 2005

15 - 55% improvements in a posteriori CO₂ flux in winter
10 - 30% improvements in summer

Importance of decreasing $\mathcal{E}_I + \mathcal{E}_R - \mathcal{E}_M$